cortex 45 (2009) 1261–1265

available at www.sciencedirect.com

ScienceDirect

journal homepage: www.elsevier.com/locate/cortex

Special section: Research report

The impact of visuo-spatial number forms on simple arithmetic

Jamie Ward[a,*], *Noam Sagiv*[b] *and Brian Butterworth*[c]

[a]*Department of Psychology, University of Sussex, Brighton, UK*
[b]*Centre for Cognition and Neuroimaging, Brunel University, UK*
[c]*Institute of Cognitive Neuroscience and Department of Psychology, University College London, UK*

ARTICLE INFO

Article history:
Received 11 November 2008
Reviewed 11 February 2009
Revised 2 March 2009
Accepted 25 March 2009
Published online 7 July 2009

Keywords:
SNARC
Synaesthesia/synesthesia
Numbers
Number form
Space

ABSTRACT

Number forms, conscious visuo-spatial representations of the sequence of numbers, are found in around 12% of the population. However, their contribution to numerical cognition is not well understood. In this study we contrast the speeded performance of individuals with number forms versus controls on single digit multiplication, subtraction and addition. Previous research has suggested that multiplication may rely more on retrieval of verbal facts whereas subtraction relies more on online calculation using a putatively spatial 'mental number line'. If people with number forms rely more heavily on visual-spatial strategies than verbal ones then we hypothesised that multiplication may be disproportionately affected by this strategy relative to subtraction, and this was found.
© 2009 Elsevier Srl. All rights reserved.

1. Introduction

It is now widely recognised that there is a spatial component to numerical cognition (Fias and Fischer, 2005; Hubbard et al., 2005). However, this spatial component is not fully understood. It is unclear to what extent its involvement is limited to certain numerical tasks (Lee and Kang, 2002), or to certain individuals (Lonnemann et al., 2008), and it is unclear whether it originates from reading direction, finger counting, or elsewhere (Wood and Fischer, 2008). One potentially important difference may lie in whether or not an individual experiences a conscious image of the sequence of numbers, also known as a 'number form' (Galton, 1880a, 1880b, 1883/1907/1973). These number forms are reported to appear automatically and take on a consistent shape over time (e.g., Sagiv et al., 2006; Seron et al., 1992). As such they share key characteristics with other types of synaesthesia (e.g., Ward and Mattingley, 2006). Although there are a large number of idiosyncrasies between individuals, there is a general trend for number forms to progress from left-to-right and for some digits (e.g., multiples of 10) to be more prominent (Sagiv et al., 2006). Despite the fact that number forms are reported in around 12% of the population (Sagiv et al., 2006), there have been very few studies that investigate how the presence of these explicit spatial representations affects numerical cognition.

In magnitude comparison tasks (i.e., deciding which of two numbers is larger or smaller), participants with number forms

* *Corresponding author*. Department of Psychology, University of Sussex, Falmer, Brighton BN1 9QH, UK.
E-mail address: jamiew@sussex.ac.uk (J. Ward).
0010-9452/$ – see front matter © 2009 Elsevier Srl. All rights reserved.
doi:10.1016/j.cortex.2009.03.017

show an effect of spatial compatibility between the visual display of digits and their number form (Piazza et al., 2006; Sagiv et al., 2006; Tang et al., 2008). For example, if the number form ascends vertically then these participants are faster at determining which number is numerically larger/smaller if the smaller number is presented below the larger number on the screen than vice versa (Jarick et al., this issue, 2009; case S5 Sagiv et al., 2006). Most participants with number forms show a left-to-right progression, but not all do. For those with number forms that go right-to-left, they are faster if the smaller number is on the right rather than the left (case S1 Sagiv et al., 2006; Piazza et al., 2006). Thus, in magnitude comparison the number-space association is determined by the number form rather than by the cultural norm. In Western cultures, the norm is for smaller numbers to be on the left and larger numbers on the right in implicit associations. This is typically demonstrated using the Spatial-Numerical Association of Response Codes (SNARC) effect in which participants are faster at making a parity judgment (odd/even) when the left hand responds to smaller numbers and the right hand responds to large numbers (Dehaene et al., 1993). In one case of a right-to-left number form, a normal SNARC effect was found (i.e., left-to-right) despite the presence of a right-to-left spatial compatibility effect for magnitude comparison (Piazza et al., 2006). This raises the possibility that the number form only exerts an influence under certain conditions (see also Hubbard et al., this issue, 2009).

A recent functional imaging study found that participants with left-to-right synaesthetic number forms showed heightened bilateral activity in a region in the posterior intraparietal sulcus (as well as various frontal locations; Tang et al., 2008). This region has been implicated in spatial aspects of numerical cognition (Dehaene et al., 2003) and lies more posteriorly to the region considered to be the 'core' area of number semantics. However, the difference in activation between those with number forms and controls was task-specific. It was found when there was spatial compatibility between the left-right number form and when deciding whether a digit was in the correct left-to-right position in a string (e.g., XXXX5). It was not found when participants with a left-right number form had to make position judgments from right to left (e.g., 5XXXX), or in a cardinality judgment task (deciding whether the digit corresponds to the number of tokens in the string).

At present, there is scant evidence for the role of number forms in arithmetic. Spalding and Zangwill (1950) report a neurological patient with penetrating head injuries to the parietal lobes who not only reported disruption to his number form but also presented with acquired dyscalculia. Questionnaire studies of people with number forms also suggest that they use them during calculation (Seron et al., 1992).[1] For example, $7 + 4$ may be realised by locating 7 on the number form and moving 4 units to the right. Similarly, Pinhas and Fischer (2008) found, in the general population, that addition was associated with rightward bias and subtraction with leftwards.

Although, evidence from people with number forms is lacking there is far more evidence for the role that spatial processes play in various aspects of arithmetic in the population at large. According to the influential model of Dehaene and Cohen (1995, 1997), addition, subtraction and multiplication make different demands on verbal versus spatial strategies. Specifically, it is argued that simple multiplication (e.g., of single digits) depends on a verbal strategy of rote retrieval of facts from a verbal memory store. In contrast, it is argued that simple subtraction is not rote-memorised but tends to be calculated online using a 'mental number line' that has a spatial component to it. Addition is assumed to rely partly on both strategies. Lee and Kang (2002) present evidence that is consistent with this view. Holding irrelevant phonological information in working memory affects multiplication but not subtraction, whereas subtraction is affected more than multiplication when irrelevant visuo-spatial information is held in mind. According to this model, how might the presence of visuo-spatial number forms affect arithmetic? One possibility is that the presence of a number form results in an over-reliance on visuo-spatial arithmetical strategies and an under-reliance on verbal strategies (assuming they have not developed alternative compensatory strategies). Seron et al. (1992) gave their participants a questionnaire (that of Paivio and Harshman, 1983) about the use of visual imagery and verbal strategies in everyday life, although not specifically arithmetic. Those with number forms did not report a greater reliance on visual strategies in everyday life relative to controls, but they did report significantly less usage of verbal strategies. If people with number forms fail to rely on verbal strategies in arithmetic then one predicts that they, relative to controls, will be more affected on multiplication relative to addition relative to subtraction. This prediction depends on the assumption that there is a normal, albeit conscious, spatial representation of number in people with number forms. An abnormality in the spatial representation of number should affect subtraction more than multiplication or addition (i.e., the opposite profile). In the study below, we test these predictions.

2. Experiment

Our previous research established that number forms are more prevalent amongst synaesthetes who experience grapheme-colour synaesthesia (Sagiv et al., 2006). In order to establish whether differences in arithmetic are due to the presence of number forms or the presence of these co-occurring types of synaesthesia, these factors were contrasted.

2.1. *Method*

2.1.1. *Participants*

Seventy-seven participants were divided into 4 groups as a 2×2 between-subjects design. One factor was the presence or absence of spatial forms (+NF and −NF), including for numbers. The second factor was the presence or absence of non-spatial synaesthesias (+syn and −syn), minimally, the non-spatial synaesthesias included colour or taste experiences induced by numbers and other verbal stimuli. Those

[1] In our sample of 154 people with number forms, 77.3% say that they use it in calculation. However, it remains to be determined which aspects of calculation it is used for or indeed whether the remaining 22.7% may use it unintentionally.

participants lacking both spatial forms and other types of synaesthesia (−syn/−NF) served as a 'normal population' control group that were recruited opportunistically from the undergraduate population and from acquaintances of the researchers ($N = 34$; mean age = 30.0, range = 18–65; 18 females). Using this sampling method, we found 6 participants (out of the initial 40) who reported number forms but not non-spatial synaesthesia (−syn/+NF) and these were treated as a separate group ($N = 6$; mean age = 30.0, range = 18–57; 5 females). From our database of known synaesthetes, we tested a group who lacked spatial forms but possessed other synaesthetic experiences, i.e., +syn/−NF ($N = 18$; mean age = 35.6, range = 20–60; 15 females) and a group of synaesthetes who possess both, i.e., +syn/+NF ($N = 19$; mean age = 38.7, range = 18–64; 15 females). In all analyses, the age and sex of participants were entered as covariates given that it was not possible to match across groups exactly. The presence of taste/colour synaesthesia was assessed by measuring the internal consistency of their colour or gustatory associations to verbal stimuli over two time periods separated by at least two weeks, and comparing each case against a group of controls by Z-score and a cut-off of $p < .05$ (for detailed method and control scores see Sagiv et al., 2006; Ward and Simner, 2005; Ward et al., 2005). The presence of a number form was by self-report on an initial questionnaire followed by interview with one of the experimenters for confirmation.

2.1.2. Materials

The stimuli consisted of 28 single digit additions (e.g., $5 + 6$); 28 single digit subtractions (e.g., $8-4$); 28 single digit multiplications (e.g., 8×4), and 28 single and double-digit numbers that served as a control. The control stimuli consisted of a representative sample of the solutions of previous problems (e.g., 11, 4, 32) that the participants were required to name.[2] This condition was included to discount any general difficulties in visual processing or naming of numbers. All answers were positive numbers (e.g., items such as 5–8 were avoided). There were no ties (e.g., $3 + 3$, 5×5) and the digits between 0 and 9 were included, with approximately the same set of digits being used in each of the three arithmetical tasks.

2.1.3. Procedure

Each type of stimuli was presented blocked together in the order: addition, subtraction, multiplication and number naming. Participants sat approximately 42 cm from a 14 inch monitor. The first four items in each series served as practice trials and were excluded from the analysis. Before each block, participants were told of the nature of stimuli and were instructed to give their response into a microphone as quickly and accurately as possible. The procedure on each trial was as follows. A fixation cross appeared in the middle of the screen for 1500 msec. Following this the stimulus appeared centrally in Times font (size 50, black on white) until a response was made. Each digit subtended a visual angle of 3° by 1.5°.

[2] The control condition was introduced after the first batch of participants had been tested, and so no data is available from 6 synaesthetes (3 in +syn/+NF and 3 in +syn/−NF) and 9 controls (−syn/−NF).

2.2. Results and discussion

Technical errors (related to the microphone) and outliers more than 3 standard deviations from the mean were removed from the analysis. Incorrect responses were removed from the response time analysis and were considered separately. A $2 \times 2 \times 3$ mixed ANOVA was conducted with the factors presence or absence of a number form (2 levels, between subjects), presence or absence of other synaesthetic experiences (2 levels, between subjects) and arithmetical task (addition, subtraction, multiplication). The dependent measure is voice-onset response times. The data is summarised in Fig. 1. There was a significant main effect of task [$F(2,138) = 9.92$, $p < .001$] suggesting that the response time in some arithmetical tasks was slower than others. There was a significant main effect of presence/absence of a number form [$F(1,71) = 6.22$, $p < .05$] and this interacted significantly with task [$F(2,142) = 3.37$, $p < .05$]. This suggests that presence of a number form does affect performance on simple arithmetical tasks, although it may affect some operations more than others. Those participants with number forms (irrespective of whether they had other types of synaesthesia) were significantly slower on addition [$t(75) = 2.64$, $p < .01$] and multiplication [$t(75) = 2.43$, $p < .05$] but not subtraction [$t(75) = 1.88$, N.S.] than those without number forms. Using Cohen's d, the effect sizes for addition, multiplication and subtraction are .61, .57 and .45. There was no main effect of non-spatial synaesthetic symptoms [$F(1,71) = .53$, N.S.] and no other interactions approached significance (all p's > .10). The covariates of age and sex were not significant and did not interact with any variables (all p's > .10). The number of incorrect responses were collapsed across the three tasks (owing to a generally low rate of errors) and analysed in a 2×2 ANOVA contrasting presence/absence of number forms and presence/absence of other synaesthetic tendencies. There were no significant main effects and no interaction (all p's > .10; average errors in the +NF groups was 4.6% and average errors in the −NF group was 4.7%). Thus, the difference in response times is unlikely to reflect a speed/accuracy trade-off. Moreover, the presence or absence of a number form had no effect on the digit naming task [$t(60) = 1.64$, $p > .10$] suggesting that these individuals are not generally slow in processing numerical stimuli.

In summary, the presence of a number form does influence performance on arithmetic and this is the first study to demonstrate this. The reasons for this are considered in the General discussion.

3. General discussion

Our results show that the presence of a number form affects arithmetical processing speed, particularly for multiplication and addition but less so for subtraction. In the general population, it is suggested that simple multiplication tends to rely on verbal rote retrieval whereas subtraction relies upon online calculation using a spatial 'mental number line', with addition relying on a mixture of both (e.g., Dehaene and Cohen, 1995, 1997; Geary et al., 1993; Cochon et al., 1999). An over-reliance on a visuo-spatial number form, and/or an under-reliance on

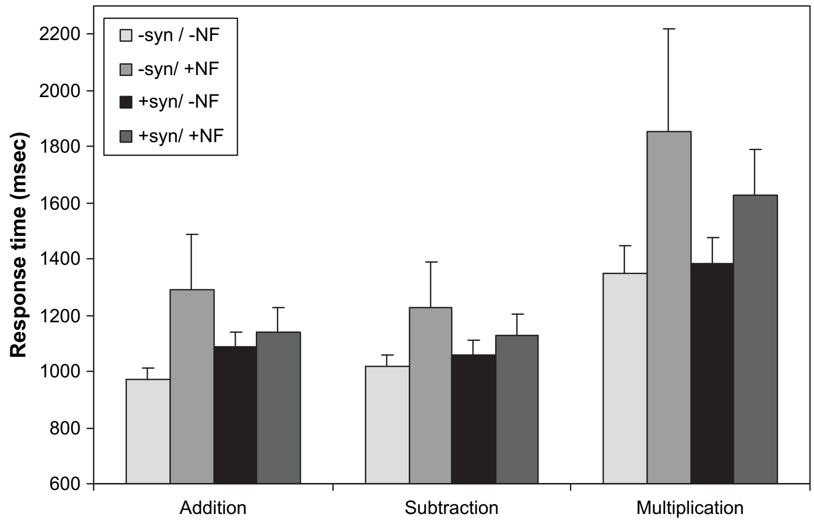

Fig. 1 – Performance on speeded arithmetic tasks depending on the presence or absence of number forms (+ NF/− NF) and the presence or absence of synaesthetic experiences of colour and/or taste (+syn/−syn). Error bars show 1 S.E.M.

the use of verbal facts, would therefore be expected to affect multiplication more than subtraction, as was found. This is consistent with previous research suggesting that those individuals who possess a number form rely significantly less on verbal strategies in everyday cognition (Seron et al., 1992). Given that this model assumes that addition could be performed either by spatial or verbal strategies, it is harder to make specific predictions about addition except for the fact that any manipulation that affects addition should also affect either subtraction or multiplication (see Van Harskamp and Cipolotti, 2001).

It is important to note that our assumption, so far, is that the spatial number form is 'normal', albeit being explicit in nature. Although it is not possible to give an a priori definition of what constitutes normality in this instance, it is to be noted that the number forms vary in complexity. Whilst some perceive the numbers as arranged linearly, others perceive bends and breaks in the number form (e.g., 1–12 arranged like a clock, or separate blocks for 10 sec, 20 sec, 30 sec etc.). This heterogeneity could be important, as more complex spatial arrangements may differ from the normally implicit (and putatively linear) arrangement in the general population. However, it is also possible that implicit spatial associations in the general population may occasionally take on forms that buck the cultural norm and may go undetected by the SNARC effect (in which the spatial positions are fixed by the location of the keyboard) but could possibly be detected with other types of spatial/motor tasks (Fischer and Campens, 2009).

It is important to consider alternative explanations of our data. First of all, we can discount the idea that the differences on arithmetic are due to synaesthesia per se. This is because non-spatial synaesthetic experiences had no significant affect on task performance. It is possible that some other difference that we didn't control for (e.g., occupation) could explain the pattern but there is no reason to think that there is an unwanted sampling bias given that participants with and without number forms were recruited in the same way. Further research is needed to test our hypothesis, for example, using verbal and spatial interference (Lee and Kang, 2002). However, we believe that the present results make an important, if preliminary, contribution in showing that number forms are not a mere epiphenomenon when it comes to arithmetic.

Even in people without number forms, there is evidence that single digit multiplication is not performed solely by verbal fact retrieval given that larger answers are retrieved more slowly than smaller ones (Ashcraft et al., 1992; Lefevre et al., 1996; Manly and Spoehr, 1999; Penner-Wilger et al., 2002). As such, tasks such as multiplication could be regarded as an outcome of different procedures with individual differences in the weighting of the strategies rather than solely relying on verbal retrieval. The presence of a number form could therefore be regarded as leading to a skew in arithmetical strategies. Given that number forms are found in as many as 12% of the adult population (Sagiv et al., 2006), our study highlights the fact that these atypical experiences should be taken into account in other studies of numerical cognition even though they are not routinely looked for.

Acknowledgements

This research was supported by a grant from the Leverhulme Trust to JW and BB.

REFERENCES

Ashcraft MH, Donley R, Halas MA, and Vakali M. Working memory, automaticity and problem difficulty. In Campbell JID (Ed), *The Nature and Origins of Mathematical Skills*. Amsterdam: North-Holland, 1992: 301–329.

Cochon F, Cohen L, van de Moortele PF, and Dehaene S. Differential contributions of the left and right inferior parietal

lobules to number processing. *Journal of Cognitive Neuroscience*, 11: 617–630, 1999.
Dehaene S, Bossini S, and Giraux P. The mental representation of parity and numerical magnitude. *Journal of Experimental Psychology: General*, 122: 371–396, 1993.
Dehaene S and Cohen L. Towards an anatomical and functional model of number processing. *Mathematical Cognition*, 1: 83–120, 1995.
Dehaene S and Cohen L. Cerebral pathways for calculation: double dissociation between rote and quantitative knowledge of arithmetic. *Cortex*, 33: 219–250, 1997.
Dehaene S, Piazza M, Pinel P, and Cohen L. Three parietal circuits for number processing. *Cognitive Neuropsychology*, 20: 487–506, 2003.
Fias W and Fischer MH. Spatial representation of number. In Campbell JID (Ed), *Handbook of Mathematical Cognition*. Hove: Psychology Press, 2005.
Fischer MH and Campens H. Pointing to numbers and grasping magnitude. *Experimental Brain Research*, 192: 149–153, 2009.
Galton F. Visualised numerals. *Nature*, 21: 252–256, 1880a.
Galton F. Visualised numerals. *The Journal of the Royal Anthropological Institute*, 10: 85–102, 1880b.
Galton F. *Inquiries into Human Faculty and its Development*. New York: Ams Press, 1883/1907/1973.
Geary DC, Frensch PA, and Wiley JG. Simple and complex mental subtraction: strategy choice and speed-of-processing differences in younger and older adults. *Psychology and Aging*, 8: 242–256, 1993.
Hubbard EM, Piazza M, Pinel P, and Dehaene S. Interactions between numbers and space in parietal cortex. *Nature Reviews Neuroscience*, 6: 435–448, 2005.
Hubbard EM, Ranzini M, Piazza M, and Dehaene S. What information is critical to elicit interference in number-form synesthesia? *Cortex*, 45: 1200–1216, 2009.
Jarick M, Dixon MJ, Maxwell EC, Nicholls MER, and Smilek D. The ups and downs (and lefts and rights) of synaesthetic number forms: validation from spatial cueing and SNARC-type tasks. *Cortex*, 45: 1190–1199, 2009.
Lee K-M and Kang S-Y. Arithmetic operation and working memory: differential suppression in dual tasks. *Cognition*, 83: B63–B68, 2002.
Lefevre J-A, Sadesky GS, and Bisanz J. Selection of procedures in mental addition: reassessing the problem size effect in adults. *Journal of Experimental Psychology: Learning, Memory and Cognition*, 22: 216–230, 1996.
Lonnemann J, Krinzinger H, Knops A, and Willmes K. Spatial representations of numbers in children and their connection with calculation abilities. *Cortex*, 44: 420–428, 2008.
Manly CF and Spoehr KT. Mental multiplication: nothing but the facts? *Memory and Cognition*, 27: 1087–1096, 1999.
Paivio A and Harshman R. Factor analysis of a questionnaire on imagery and verbal habits and skills. *Canadian Journal of Psychology*, 37: 461–483, 1983.
Penner-Wilger M, Leth-Steensen C, and LeFevre JA. Decomposing the problem-size effect: a comparison of response time distributions across cultures. *Memory and Cognition*, 30: 1160–1167, 2002.
Piazza M, Pinel P, and Dehaene S. Objective correlates of an unusual subjective experience: a single-case study of number-form synaesthesia. *Cognitive Neuropsychology*, 23: 1162–1173, 2006.
Pinhas M and Fischer MH. Mental movements without magnitude? A study of spatial biases in symbolic arithmetic. *Cognition*, 109: 408–415, 2008.
Sagiv N, Simner J, Collins J, Butterworth B, and Ward J. What is the relationship between synaesthesia and visuo-spatial number forms? *Cognition*, 101: 114–128, 2006.
Seron X, Pesenti M, Noel M-P, Deloche G, and Cornet JA. Images of numbers, or ''When 98 is upper left and 6 sky blue''. *Cognition*, 44: 159–196, 1992.
Spalding JMK and Zangwill OL. Disturbance of number-form in a case of brain injury. *Journal of Neurology, Neurosurgery and Psychiatry*, 13: 24–29, 1950.
Tang J, Ward J, and Butterworth B. Number forms in the brain. *Journal of Cognitive Neuroscience*, 2008.
Van Harskamp NJ and Cipolotti L. Selective impairments for addition, subtraction and multiplication: implications for the organisation of arithmetical facts. *Cortex*, 37: 363–388, 2001.
Ward J and Mattingley JB. Synaesthesia: an overview of contemporary findings and controversies. *Cortex*, 42: 129–136, 2006.
Ward J and Simner J. Is synaesthesia an x-linked dominant trait with lethality in males? *Perception*, 34: 611–623, 2005.
Ward J, Simner J, and Auyeung V. A comparison of lexical-gustatory and grapheme-colour synaesthesia. *Cognitive Neuropsychology*, 22: 28–41, 2005.
Wood G and Fischer MH. Numbers, space, and action: from finger counting to the mental number line and beyond. *Cortex*, 44: 353–358, 2008.